NUMERICAL ANALYSIS

Assignment -1 (week 1)

Total Marks - 25

Posted on - 28/7/2017 (Friday);

To be submitted on or before - 6/8/2017 (Sunday), 23.59 hours.

Problems on

- Approximation of $f(x)$ using Taylor's theorem with Lagrange form of remainder
- Absolute and Relative error

INSTRUCTIONS

- This is a question paper cum answer booklet.
- Take a print out of this.
- Present the details of the computations of the solution of each problem **which you will have to show** in the space provided at the bottom of the page.
- Fill in the answers in the space provided below each question.
- Scan the booklet and submit it as a pdf file before the deadline for evaluation.

1. Let $f(x) = \sqrt{x+1}$. Use the third degree Taylor polynomial $p_3(x)$ about $x = 0$ of $f(x)$ to approximate $\sqrt{0.75}$. Find the absolute error.
Fill in the blanks:
(a) If $p_3(x) = A + Bx + Cx^2 + Dx^3$, then

$A =$ _____

$B =$ _____

$C =$ _____

$D =$ _____

(b) $p_3(-0.25) =$ _____

(c)|error| $= |p_3(-0.25) - \sqrt{0.75}| \simeq$ _____. (6 marks)

Show your work for the solution of problem 1 in the space provided below.

2. Let $R_4(x)$ be the remainder term of the Lagrange form in the approximation of e^x by the Taylor polynomial $p_4(x)$ of degree 4, about $x = 0$. Estimate the error in approximating $e^{-0.5}$ by $p_4(-0.5)$. Use your calculator to determine the actual absolute arror for the estimate $p_4(-0.5)$ of $e^{-0.5}$.
Fill in the blanks:
(a)$|R_4(-0.5)| \leq$ _____

(b) the absolute error $= |p_4(-0.5) - e^{-0.5}| \simeq$ _____. (4 marks)

Show your work for the solution of problem 2 in the space provided below.

3. A linear interpolation is used to estimate $f(11.1)$ where $f(x)$ is known at a set of equally spaced points as given below:

x	10	11	12
y	64	65	69

Obtain the estimate of $f(x)$ at $x = 11.1$.
Fill in the blank:
$f(11.1) \simeq p_1(11.1) =$ _____. (2 marks)

Show your work for the solution of problem 3 in the space provided below.

4. Eestimate $ln(0.54)$ using Newton's forward difference interpolation polynomial p_4 for the data

x	0.4	0.5	0.6	0.7	0.8
$f(x)$	-0.916291	-0.693147	-0.510826	-0.356675	-0.223144

Fill in the blanks:
(a) The fourth order forward difference $\triangle^4 f(x_0) =$ _____.

(b) $ln(0.54) \simeq p_4(0.54) =$ _____. (8 marks)

Show your work for the solution of problem 4 in the space provided below.

5. If $x = 0.3721478693$ and $y = 0.3720230572$, what is the relative error in the computation of $x - y$ using five decimal digits of accuracy.
Fill in the blanks:
(a) $x - y =$ _____.

(b) If x and y are rounded to five decimal digits x^* and y^* respectively, then
$x^* =$ _____,
$y^* =$ _____,
$x^* - y^* =$ _____.
(c) Relative error = _____. (5 marks)

Show your work for the solution of problem 5 in the space provided below.